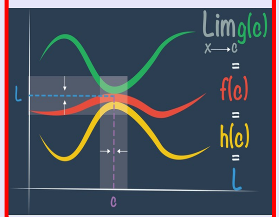
Calculus I

Lecture 33



Feb 19-8:47 AM

How do we use first derivative?

If $f'(x) > 0 \to f(x)$ is increasing.

If $f'(x) < 0 \to f(x)$ is decreasing.

If $f'(x) = 0 \to f(x)$ has horizontal tan. line. It may have Max. or min. Point.

When $f'(x) = 0$ or $f'(x)$ is undefined, we have Critical Points.

$f(x) = x^2 - 4x + 3$

$f'(x) = 2x - 4$

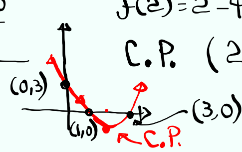
$f'(x) = 0$

$2x - 4 = 0 \to \boxed{x = 2}$

$f(2) = 2^2 - 4(2) + 3 = \boxed{-1}$

C.P. $(2, -1)$

x	$-\infty$	2	∞
$f'(x)$	$-$		$+$
$f(x)$	$\searrow$		$\nearrow$



Oct 28-7:28 AM

$f(x) = \dfrac{x}{x-1}$

Y-Int $(0,0)$

$f(0) = \dfrac{0}{0-1} = \dfrac{0}{-1} = 0$

x-Int $(0,0)$

$f(x) = 0$

$\dfrac{x}{x-1} = 0$

$x = 0$

Domain $(-\infty, 1) \cup (1, \infty)$

V.A. $x = 1$

$\lim\limits_{x \to \infty} \dfrac{x}{x-1} = \lim\limits_{x \to \infty} \dfrac{\frac{x}{x}}{\frac{x}{x} - \frac{1}{x}} = \lim\limits_{x \to \infty} \dfrac{1}{1 - \frac{1}{x}} = 1 \leftarrow$ H.A.

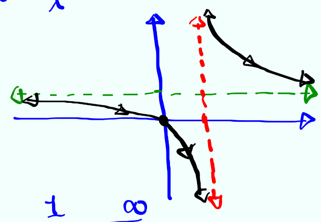
$f(x) = \dfrac{x-1+1}{x-1} = \dfrac{x-1}{x-1} + \dfrac{1}{x-1}$

$f(x) = 1 + (x-1)^{-1}$

$f'(x) = 0 - 1(x-1)^{-2} \cdot 1$

$f'(x) = \dfrac{-1}{(x-1)^2} < 0$

$f'(x) \neq 0$, $f'(x)$ is undefined at $x = 1$



x	$-\infty$	1	∞
$f'(x)$	$-$		$-$
$f(x)$	$\searrow$		$\searrow$

Oct 28-7:37 AM

$f(x) = 1 + \sin x$, $0 \leq x \leq 2\pi$

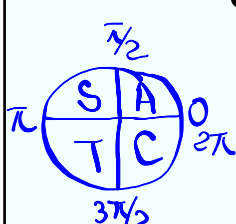
$f'(x) = 0 + \cos x$

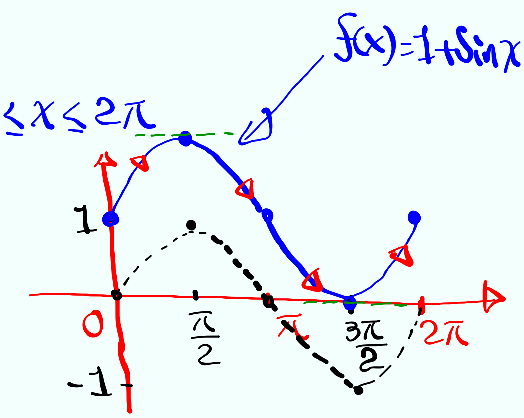
$f'(x) = \cos x$

C.P. $\rightarrow f'(x) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$





x	0		$\frac{\pi}{2}$		$\frac{3\pi}{2}$		2π
$f'(x)$		$+$		$-$		$+$	
$f(x)$		$\nearrow$	Max	$\searrow$	Min	$\nearrow$	

Oct 28-7:46 AM

How do we use the Second derivative?

If $f''(x) > 0 \to f(x)$ is Concave up.

If $f''(x) < 0 \to f(x)$ is Concave down.

when $f''(x) = 0$ or $f''(x)$ is undefined, we have possible inflection Point.

what is an inflection Point?

Inflection Point is where Concavity changes.

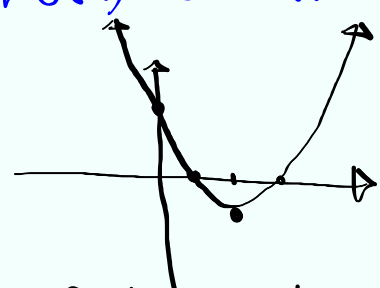
Oct 28-7:55 AM

$f(x) = x^2 - 4x + 3$

$f'(x) = 2x - 4 \to$ C.P. $(2, -1)$

$f''(x) = 2$, $f''(x) > 0 \to f(x)$ is Concave Up.

x	$-\infty$	2	∞
$f'(x)$	$-$	$\bullet$	$+$
$f''(x)$	$+$		$+$
$f(x)$	$\smile$		$\smile$

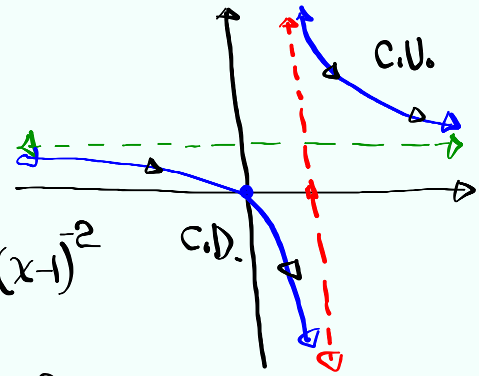


No inflection Point.

Oct 28-7:59 AM

$f(x) = \dfrac{x}{x-1}$

See earlier work



$f'(x) = \dfrac{-1}{(x-1)^2}$ $\quad f'(x) = -1(x-1)^{-2}$

$f''(x) = -1\cdot(-2)\cdot(x-1)^{-3}\cdot 1 = \dfrac{2}{(x-1)^3}$

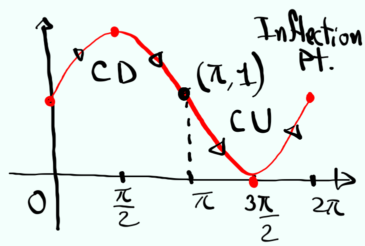
x	$-\infty$	1	∞
$f'(x)$	$-$	o	$-$
$f''(x)$	$-$	o	$+$
$f(x)$	↘	P.I.P.	↘

Concavity changed at $x=1$, however it is not an inflection Point

Oct 28-8:04 AM

$f(x) = 1 + \sin x$, $0 \leq x \leq 2\pi$

See earlier work



$f'(x) = \cos x \rightarrow$ C.P. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f''(x) = -\sin x \rightarrow$ P.I.P. $-\sin x = 0$

$\sin x = 0$, $\boxed{x=0}$, $x=\pi$, $\boxed{x=2\pi}$

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$f'(x)$		$+$	•	$-$		$-$	•	$+$	
$f''(x)$		$-$		$-$	•	$+$		$+$	
$f(x)$		↗	Max	↘	I.P.	↘	Min.	↗	

Oct 28-8:12 AM